Geometric Algorithms

Final exam, August 30, 2007

9.00–12.00 am

Credits: You get 10 points for free. The maximum number of credits is indicated for each of the four assignments. Explain clearly the correctness of your solution, and give a derivation of the complexity of your algorithm. Avoid long pointless arguments. Correct answers with sloppy or incomplete presentations only deserve partial credits.

Use of literature: You may use Edelsbrunner's book and the notes on Computational Topology, as well as one sheet with mathematical formulas.

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1. Farthest Point Voronoi Diagram (20)
Let S be a set of points in the plane. The Farthest Voronoi Cell $FPV(p)$ of $p \in S$ is the set of points x such that $\| x - p \| \geq \| x - q \|$, for all $q \in S$.

1. Prove that $FPV(p)$, $p \in S$, is a convex polygon (possibly unbounded).
2. Prove that $FPV(p)$, $p \in S$, is non-empty iff p is an extremal point of S.

2. Checking the Delaunay property (20)
Let S be a set of n points in the plane, and let $\mathcal{T}$ be a triangulation of the convex hull of S. Give an algorithm that checks in $O(n)$ time whether $\mathcal{T}$ is a Delaunay triangulation of S.

3. Weighted Voronoi Diagrams (25)
Given a set S of n weighted points in the plane. Recall that an extremal point of S is nonredundant (irrespective of the choice of the weights).

1. Prove that we can choose the weights in such a way that every non-extremal point is redundant.
2. Give an algorithm that computes such a set of weights in $O(n \log n)$ time.

4. Betti numbers (25)
1. Compute the Betti numbers of a disk in the plane with h holes ($h \geq 0$).

2. Let M be a surface of genus g (a 'sphere with g handles'). Prove that a Morse function on M has at least $2 + 2g$ critical points.
